A Distributed Cooperative Search Algorithm using Multiple Contexts and Pruning

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Abstract

Distributed constraint optimization problems (DCOPs) have been studied as a fundamental framework of cooperative problem solving in multiagent systems. Exact solvers based on tree search and dynamic programming have been proposed for DCOPs. Based on tree searches, the solvers perform iterative processing that depends on message communication. Therefore the overhead of the communication affects the execution time of the search. On the other hand, solvers that are completely based on dynamic programming require no iterative processing among agents. However, its memory and message size is exponential to the induced width of pseudo-trees. Although memory-bounded solvers that employ both methods have been proposed, several solvers search for one solution at a time. Other solvers employ no pruning based on global cost values. Especially, when the communication overhead is relatively large, simultaneously expanding multiple solutions and transferring them by the same message are reasonable. In this study, we present the basic algorithms of memory-bounded solvers that employ multiple search points.

1 Introduction

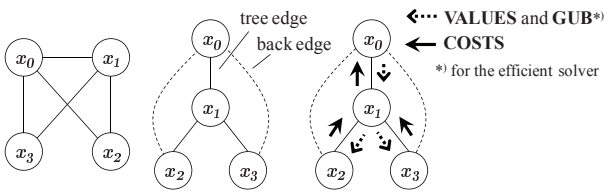
Distributed constraint optimization problems (DCOPs) [2, 3, 4, 10] have been studied as a basic framework of cooperative problem solving in multiagent systems. With DCOPs, the states of agents and the relationships between agents are formalized into a constraint optimization problem. The problem is solved by distributed search algorithms. These studies focus on the optimization problems and the distributed search algorithms that are essentially contained in cooperative protocols of the multiagent systems. Several cooperative problems including distributed resource scheduling and sensor networks are represented as DCOPs [1, 10]. Exact solvers based on tree search and dynamic programming have been proposed for DCOPs. Based on tree searches, the solvers perform iterative processing that depends on message communication. Therefore the overhead of the communication affects the execution time of the search. On the other hand, solvers that are completely based on dynamic programming [4] require no iterative processing among agents. However, memory and message size is exponential to the induced width of the pseudo-trees. Although memory-bounded solvers that employ both methods have been proposed, several solvers search for one solution at a time [3]. Other solvers employ no pruning based on global cost values [5, 6]. Especially, when the communication overhead is relatively large, simultaneously expanding multiple solutions and transferring them by the same message are reasonable. In this study, we present the basic algorithms of memory-bounded solvers that employ multiple search points. Such methods resemble the execution of overlapped multiple search processes. Basically, the proposed methods are generalizations of previous studies. In search processing, the solver recomposes the solution sets that are currently expanded. The solution spaces of the sub-problems are considered to recompose a set of solutions. For the set of solutions, boundaries of the global cost values are simultaneously computed. By the multiple search points and the pruning based on the boundaries reduce iterative search among agents. The algorithm can also be considered as the basis of solvers that employ a mixed search strategy.

The rest of our paper is organized as follows. In Section 2, the background of the study is shown. Our proposed methods are shown in Section 3, 4 and 5. The methods are experimentally evaluated in Section 6. In Section 7, we conclude our study.

2 Backgrounds

2.1 Distributed constraint optimization problem

In this study, we use a fundamental definition of distributed constraint optimization problems. A problem is defined by set A of agents, set X of variables, set C of binary constraints, and set F of binary functions. Agent i has its own variable x_i that takes a value from discrete finite domain D_i. The value of x_i is controlled by agent i. Constraint $c_{i,j}$ represents the relationship between x_i and x_j. The cost of assignment $\{(x_i, d_i), (x_j, d_j)\}$ is defined by binary function



(a) constraint network (b) pseudo-tree (c) message paths

Figure 1: Pseudo-tree and messages

$f_{i,j}(d_i, d_j) : D_i \times D_j \rightarrow \mathbb{N}_0$. The goal is to find global optimal solution $\mathcal{A}$ that minimizes the global cost function: $\sum_{f_{i,j} \in F,\ \{(x_i,d_i),(x_j,d_j)\} \subseteq \mathcal{A}} f_{i,j}(d_i, d_j)$. Agent i knows the constraints and the cost functions that are related to x_i. The search process to find the optimal solution is represented as a distributed algorithm based on message communication between agents.

2.2 Pseudo-trees

A pseudo-tree [4, 7], which is a graph structure that defines a partial order on variables, is based on a spanning tree of the constraint network. A typical pseudo-tree is generated using a depth-first traversal of a constraint network. For example, the pseudo-tree in Figure 1 (b) is generated from the constraint network in Figure 1 (a). In the pseudo-tree, the edges of the original constraint network are categorized into either tree edges or back edges. The tree edges are the edges of the spanning tree. The other edges are back edges. The tree edges represent the partial order relation between the two variables. We consider the tree edges of the pseudo-tree the edges of the corresponding spanning tree. Also, vertices, variables, and agents may not be strictly distinguished. The following notations are used:

$prnt_i$: parent variable of x_i.
$Chld_i$: set of child variables of x_i.
Nbr_i^u: partial set of ancestor variables of x_i. The variables in Nbr_i^u are related to x_i by constraints. The variables are called pseudo-parents.
Ast_i: partial set of ancestor variables of x_i. Let x_k denote a variable in Ast_i. For at least one variable x_j that is contained in the pseudo-tree rooted at x_i, x_k has relationship $x_k \in Nbr_j^u$.

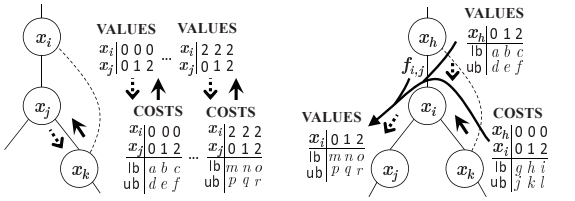
No back edge exists between different sub-trees. By employing this property, search processing can be performed in parallel.

2.3 Computation of cost values

We outline cost computation using pseudo-trees [3, 4]. Below, we assume that agents have already received both variables' values and cost values from other agents. Agent i's computation is based on partial solution s_i of Ast_i. s_i is called *context*.

Local cost $\delta_i(s_i \cup \{(x_i, d)\})$ for context s_i and value d of variable x_i is defined as follows.

$$\delta_i(s_i \cup \{(x_i, d)\}) = \sum_{(x_j,d_j) \in s_i,\ j \in Nbr_i^u} f_{i,j}(d, d_j) \qquad (1)$$



(a) expanding multiple solutions (b) aggregation of top-down cost for x_j

Figure 2: Computation of cost value

Optimal cost $g^*(s_i)$ for context s_i and the sub-tree routed at x_i are recursively defined as follows.

$$g_i^*(s_i) = \min_{d \in D_i} g_i(s_i \cup \{(x_i, d)\}) \qquad (2)$$

$$g_i(s_i \cup \{(x_i, d)\}) = \delta_i(s_i \cup \{(x_i, d)\}) + \sum_{j \in Chld_i} g_j^*(s_j) \quad \text{s.t.} \quad s_j \subseteq (s_i \cup \{(x_i, d)\}) \qquad (3)$$

In actual computation, some cost values may remain unknown. In such cases, the upper and lower limit values are used as default values. In this work, we use 0 and ∞ as the lower and upper limits. These default values separate a cost value into upper and lower bound values.

When globally optimal cost $g_r^*(\phi)$ is computed for root variable x_r, r determines the optimal assignment of its variable. Similarly, an optimal solution for the rest of the problem can be computed in a top-down manner.

2.4 Previous works

A number of exact search methods of DCOPs are performed based on the pseudo-trees. These algorithms are based on dynamic programming and branch-and-bound. Dynamic programming [4] computes the optimal cost values of sub-trees from leaf agents to a root agent. Then the optimal assignments are decided from a root agent to leaf agents. There is no iterative processing. However, the size of the memory and the messages are exponential to induced width of the pseudo-trees because each agent i simultaneously computes $g^*(s_i)$ for all assignments of the variables contained in Ast_i. Although computation of the optimal cost values can be divided into iterative processing [5, 6], pruning methods that employ global cost values have not been discussed. In memory-bounded methods based on dynamic programming and branch-and-bound [3], one current context s_i is processed in each agent i. If the overhead of the message communication is relatively large, it is reasonable to expand multiple solutions at the same time and to send them in one message because the number of iterations that contain the message communication is reduced. In previous studies, several efficient methods have been proposed [8, 9]. However, the methods that simultaneously expand multiple solutions have not been adequately discussed.

3 Multiple contexts search — basic solver

In this study, we generalize the exact methods of DCOPs to handle multiple current contexts. As a basic

method, we present an algorithm that does not employ pruning based on global cost values. The computation essentially resembles a memory bounded dynamic programming [6]. The method employs two types of messages (VALUES and COSTS in Figure 1(c)). As shown in Figure 2(a), multiple contexts are expanded using VALUES messages that are propagated based on the tree edges of a pseudo-tree in a top-down manner. In this example, three contexts are simultaneously expanded. On the other hand, cost values for current contexts are computed using COSTS messages that are propagated in a bottom-up manner. The processing is repeated until the upper and lower bounds of the global cost values reach identical values in the root agent. Then the optimal assignment of root variable is determined. Similarly, the optimal solution for the rest of the problem is recursively determined. In the following, we describe the components of the algorithm and show several explanations about important points.

Agent i uses following the variables and the data structures in addition to the representations of the DCOPs and the pseudo-trees:

S_i: a set of current contexts for the variables contained in Ast_i. The contexts are received from the parent of i.

U_i: a set that represents a table of the cost values for S_i. Element u of U_i consists of ($a(u)$, $lb(u)$, $ub(u)$). $a(u)$ is a partial solution. $ub(u)$ and $lb(u)$ represent the upper and lower bounds of the cost value of $a(u)$. U_i is sent to the parent of i.

$sAct_i$: a state of S_i. When new contexts are received from the parent of i, $sAct_i$ is set to true. As mentioned below, when closed(U_i) is true, $sAct_i$ is set to false.

$sOpt_i$: a state of S_i, which is received from parent of i. When S_i contains an optimal solution, $sOpt_i$ is true. Otherwise, it is false.

opt_i: a state of i. When the optimal assignment of x_i has been determined, opt_i is true. Otherwise, it is false.

D_i^*: a domain that contains only one optimal assignment of x_i.

R_j^i: a set of contexts for Ast_j of child j, which is computed from S_i. Let R_i denote set $\{s' | s' = s \cup \{(x_i, d)\}, s \in S_i, d \in D_i\}\}$. An element of R_j^i is recomposed from an element of R_i by eliminating the assignments of variables that are not contained in Ast_j. An element of R_j^i corresponds to at least one element of R_i.

S_j^i: a subset of R_j^i, which is sent to child j as a set of current contexts.

U_j^i: a set that represents a table of the cost values for R_j^i. The elements of U_j^i resemble elements of U_i and are updated using U_j received from child j.

$sFil_j^i$: If all S_j^i have been sent to child j for all elements of R_j^i, $sFil_j^i$ is true. Otherwise, it is false.

The messages transfer the following information:

VALUES: agent i sends S_j^i and $sOpt_i$ to child j using VALUES messages.

COSTS: agent i sends U_i that corresponds to S_i, i.e. S_i^j, to parent j by COSTS messages.

```
1  Main(){
2   Initialize().
3   until(forever){
4    until(receive loop is broken){ receive messages. }
5    if(S_i ≠ φ ∧ sAct_i){ Maintenance(). } } }
6  Initialize(){
7   S_i ← φ. sAct_i ←false. sOpt_i ←false. opt_i ←false.
        D_i^* ← φ.
8   foreach j s.t. x_j ∈ Chld_i{
9    (S_j^i, U_j^i, R_j^i, sFil_j^i)←(φ, φ, φ, false). }
10  if(x_i is root){ send (VALUES, {φ}, true) to i. } }
11 Receive(VALUES, S, sOpt){
12  S_i ← S. sAct_i ←true. sOpt_i ← sOpt.
13  foreach j s.t. x_j ∈ Chld_i{
14   reset child states (S_j^i, U_j^i, R_j^i, sFil_j^i). } }
15 Receive(COSTS, j, U){
16  if(compatible(U,S_j^i)){ merge U to U_j^i. } }
17 Maintenance(){
18  update U_i and test optimal condition.
19  if(opt_i ∧ (D_i has not been replaced by D_i^*)){
20   replace D_i by D_i^*.
21   foreach j s.t. x_j ∈ Chld_i{
22    reset child states (S_j^i, U_j^i, R_j^i, sFil_j^i). }
23   update U_i and test optimal condition. }
24  MaintainAndSendChildrensContexts().
25  if(x_i is not root){ send (COSTS,i,U_i) to i s.t. x_i = prnt_i. }
26  if(closed(U_i)){ sAct_i ←false. } }
27 MaintainAndSendChildrensContexts(){
28  foreach j s.t. x_j ∈ Chld_i{
29   if(¬(contexts of j are active) ∧ ¬sFil_j^i){
30    set next contexts S_j^i.
31    if(S_j^i is last part of R_j^i){sFil_j^i ←true.}
32    send(VALUES,S_j^i, opt_i) to j. } } }
```

Figure 3: Multiple contexts search

3.1 Reduced contexts in branching

Agent i sends the current sets of contexts to its child j using VALUES messages. The current set of contexts is a partial set of R_j^i that is a set of partial solutions for variables contained in Ast_j. Assignments of other variables are eliminated because the cost values of the subtree rooted at j only depend on Ast_j. This reduces the size and the number of partial solutions to be expanded. Additionally, the assignments of x_i's ancestor variables that are not contained in Ast_i are not leaked to agent i. When the cost values of the current context converge, the contexts S_j^i are updated to the next part of R_j^i that are sub-problems of S_i.

When opt_i is false (*i.e.* the optimal assignments have not been determined), S_j^i is updated as follows. Let R_j^{i+} denote set $R_j^i \backslash \{a(u) | u \in U_j^i\}$. S_j^i is updated to satisfy condition $S_j^i \subseteq R_j^{i+} \wedge |S_j^i| \leq L$. Here L is the maximum number of current contexts. Then initial values $\{u | a(u) \in S_j^i, lb(u) = 0, ub(u) = \infty\}$ are added to U_j^i. On the other hand, when opt_i is true, S_j^i is updated so that S_j^i contains

only one optimal element. If U_j^i contains no element u such that $a(u) = r$ for $r \in R_j^i$, $(r, 0, \infty)$ is added to U_j^i.

3.2 Asynchronous computation of cost values

Agent i receives the upper and lower bounds U_j of the cost values of R_j^i from its child j using COSTS messages. Then the cost values are stored in U_j^i. For current contexts S_i, cost table U_i of the subtree rooted at i is computed based on all U_j^i and the local costs of i. For all elements s of S_i, U_i's elements u_s such that $u_s = (s, lb(u_s), ub(u_s))$ are computed. $lb(u_s)$ and $ub(u_s)$ are computed based on the computation of g_i^* shown in Expressions 2 and 3. First, for each d in D_i, lower bound cost $lb_{s\cup\{(x_i,d)\}}$ of $s \cup \{(x_i, d)\}$ is computed using local cost $\delta_i(s \cup \{(x_i, d)\})$ and all $lb(u')$ such that $a(u') \subseteq s \cup \{(x_i, d)\}, u' \in U_j^i, x_j \in Chld_i$. Then $lb(u_s)$ is computed as $\min_{d\in D_i} lb_{s\cup\{(x_i,d)\}}$. Similarly, $ub(u_s)$ is computed using upper bound cost values. If ($s(u')$, $lb(u')$, $ub(u')$) are not contained in U_j^i, the values of $lb(u')$ and $ub(u')$ are 0 and ∞.

i repeatedly sends COSTS messages to its parent until $lb(u) = ub(u)$ for all elements u of U_i. Note that the boundaries $lb(u)$ and $ub(u)$ cannot be identical in the first step of the computation because they are partially computed. On the other hand, when the boundaries of the cost values are converged in all U_j^i's elements that corresponds to S_j^i, agent i asynchronously updates S_j^i to the next set of contexts. Then S_j^i is sent to j by a VALUES message to i's children. Even if COST messages for the previous contexts are received from the children, the old messages are ignored.

3.3 Details of the basic solver

Details of the basic solver is shown in Figure 3. The following are the procedures and functions in the figure:

reset child states (S_j^i, U_j^i, R_j^i, $sFil_j^i$): For child j, (S_j^i, U_j^i, $sFil_j^i$)←(ϕ, ϕ, false). Then R_j^i is updated.

compatible(U,S): if S contains $a(u)$ for all u in U, the return value is true. Otherwise, the return value is false.

merge U to U': For all elements u of U, U''s elements u' such that $a(u') = a(u)$ are updated as follows. $lb(u') \leftarrow \max(lb(u'), lb(u))$. $ub(u') \leftarrow \min(ub(u'), ub(u))$.

update U_i and test optimal condition: U_i is updated as shown in Section 3.2. In the computation of U_i, a condition of the optimal solution is also tested. If $sOpt_i \wedge (\exists u \in U_i, lb(u) = ub(u)) \wedge \neg opt_i$, then i determines the optimal solution as follows: $opt_i \leftarrow true$, $D_i^* \leftarrow \{d^*\}$. In the above, the upper bound cost of $a(u) \cup \{(x_i, d^*)\}$ equals to $lb(u)$ and $ub(u)$. The algorithm selects one optimal solution. Therefore, when $sOpt_i$ is true, $|S_i| = |U_i| = 1$.

closed(U_i): If, for all u in U_i, $lb(u) = ub(u)$, then the return value is true. Otherwise, the return value is false.

set next contexts S_j^i: S_j^i is updated to the next contexts as shown in Section 3.1.

contexts of j are active: For at least one element s of S_j^i and U_j^i's element u such that $a(u) = s$, if $lb(u) \neq ub(u)$, then the return value is true. Otherwise, the return value is false.

4 Pruning and reusing cost values

In this section, we apply efficient methods to the basic algorithm shown in Section 3. The most important point is the pruning based on global cost values. To compute the upper and lower bounds of the global cost values, the cost values of ancestors and other subtrees are computed in a top-down manner as shown in Figure 2(b). In this example, cost values of the partial network except for the subtree rooted at x_j are aggregated via x_i. Now VALUE messages transfer cost values of contexts. As the summations of the top-down costs of x_j and the costs of x_j's subtree, the global upper and lower bounds of cost values are obtained. The global upper bounds improve the best cost value. On the other hand, the global lower bounds are evaluated to prune partial solutions. New top-down message GUB is introduced to propagate the globally best cost value(Figure 1(c)). Moreover, the computation results are reused to reduce redundant searches. Below we mainly describe the difference from the basic methods. The variables and the data structures of agent i are modified as follows:

S_i: Each partial solution is integrated with the upper and lower bounds of the cost values that are computed in a top-down manner. Similar to U_i, element s of S_i consists of ($a(s)$,$lb(s)$,$ub(s)$). $a(s)$ is a partial solution. $ub(s)$ and $lb(s)$ are upper and lower bounds of cost values for ancestors and other subtrees.

R_j^i: Similar to S_i, the elements of R_j^i consist of a partial solution and the related upper and lower bounds. The $lb(r)$ of R_j^i's element r is computed as a summation of the following cost values:

- $lb(s)$ of S_i's element s such that $(a(r)\backslash\{(x_i, d)\}) \subseteq s$.
- local cost $\delta_i(s \cup \{(x_i, d)\})$.
- all $lb(u')$ such that $a(u') \subseteq s \cup \{(x_i, d)\}, u' \in U_k^i, x_k \in Chld_i\backslash\{x_j\}$.

Similarly, $ub(r)$ is computed using the upper bounds of the cost values. S_i can contain multiple elements s such that $(a(r)\backslash\{(x_i, d)\}) \subseteq s$. In such cases, minimum $lb(r)$ and maximum ub(r) are used as $lb(r)$ and ub(r).

S_j^i: The elements of S_j^i is resemble the elements of S_i and R_j^i.

U_j^i: U_j^i represents the upper and lower bound costs for R_j^i, which is also employed as a cache memory. When S_i is updated to different set of contexts, the elements of U_j^i are not immediately erased. If element u of U_j^i corresponds to the element of current R_j^i (for S_i), then u is reused. If $|U_j^i| \leq K$ is not satisfied when a new element is added to U_j^i, then an element is erased based on LRU. Here, $K \geq |R_j^i|$.

In addition, the following elements are employed:

gUB_i: Upper bound of global cost values. gUB_i represents the currently minimal cost value of complete solutions.

$sCls_i$: a state of S_i. $sCls_i$ represents the value of cls_j of parent j.

cls_i: represents that the upper and lower bounds of the top-down costs are closed in i. If $sCls_i$ is true and $lb(s) = ub(s)$ is true for all children j and all elements s of S^i_j, then its value is true. Otherwise, its value is false.

$sChg^i_j$: represents that S_i or the state of S_i have been just changed when its value is true.

$sFin_i$: a state of S_i. The initial value of $sFin_i$ is false. When root agent r determines the optimal assignment of x_r, $sFin_r$ is set to true. Then the value of $sFin_i$ is propagated to all agents. This changes the pruning condition.

There are no modifications in U_i, $sOpt_i$, opt_i, D^*_i, $sAct_i$, $sFil^i_j$. The following are the modifications of the messages:

VALUES: The structures of S^i_j and the related states are modified. VALUES messages that concern current S^i_j are repeatedly sent until the boundaries of the costs of S^i_j are closed.

GUB: The value of gUB_i is sent to the children by the GUB messages.

4.1 Top-down computation of cost values

In this method, the top-down computation of the cost values for ancestors and other subtrees, and the bottom-up computation of the optimal cost are performed at the same time. By combining the top-down and bottom-up costs, more precise boundaries of the global costs can be computed. To compute the top-down costs, elements s of current contexts S_i that are sent from i's parent to agent i consist of partial solution $a(s)$, upper bound $ub(s)$, and lower bound $lb(s)$ of the cost value. The cost value of each element of R^i_j that is a set of partial solutions for child j of agent i is computed by adding the costs of S_i, the local cost of i, and the costs of U^i_k for all children k except for the child j (Figure 2(b)). Note that the multiple elements of S_i can correspond to an element of R^i_j when the assignments of the contexts are reduced as shown in Section 3.1. In such case, the widest boundaries are used so that the boundaries do not exceed the true cost value.

Current set S^i_j of the contexts for R^i_j is sent from i to j by VALUES messages. If there are other subtrees of agent i, the boundaries of the subtrees' costs may not be closed immediately after S_i is updated. Therefore, until the boundaries are closed, the VALUES messages are repeatedly sent. The top-down computation depends on current S_i. However, the computation is independent from the bottom-up computation of the optimal cost. Its convergence is managed using cls_i and $sCls_i$.

4.2 Pruning using global cost values

Agent i minimizes upper bound gUB_i of the global cost value. gUB_i is sent to the other agents by GUB messages. By comparing gUB_i and the lower bound of the costs of the current solution, i judges whether the search can be pruned. Pruned partial solutions are not sent to child agents. When the optimal assignment of the root variable has not been determined yet, the search is pruned if the lower bound exceeds or equals to the global upper bound. After the optimal assignment of the root variable is determined, the search is pruned if the lower bound exceeds the global upper bound. To switch the pruning condition, $sFin_i$ is propagated.

4.3 Reusing cost values

Agent i stores the information of the cost values received from child j into U^i_j. When current contexts S_i is updated, elements u of U^i_j are not immediately erased. To limit $|U^i_j|$, the elements of U^i_j are managed by LRU. However, u, which corresponds to R^i_j (and current S_i), must not be erased from U^i_j while the costs for S_i are being computed. Therefore, to avoid erasing these elements, when S_i is updated, the reusable elements of U^i_j are moved to the end of the LRU queue.

4.4 Details of the efficient solver

Details of the efficient algorithm is shown in Figure 4. The following are the procedures and functions in the figure:

reset child states (S^i_j, U^i_j, R^i_j, $sFil^i_j$, $sChg^i_j$): For child j, (S^i_j, $sFil^i_j$, $sChg^i_j$)$\leftarrow$(ϕ, false, false). Then R^i_j is updated based on current S_i and D_j. All U^i_j's elements u such that $\exists r, r \in R^i_j, a(r) = a(u)$ are moved to the end of the LRU queue.

compatible(U,S): If S contains s such that $a(s) = a(u)$ for all elements u of U, the return value is true. Otherwise, the return value is false.

update (U_i, gUB_i) and test optimal condition: There are no modifications in the computation of U_i. Upper bound gub_s of the global cost value is computed for each element s of S_i. First, for each s and $d \in D_i$, upper bound $gub_{a(s)\cup\{(x_i,d)\}}$ is computed as a summation of the following cost values:

- $ub(s)$.
- local cost $\delta_i(a(s) \cup \{(x_i, d)\})$.
- all $ub(u')$ such that $a(u') \subseteq a(s) \cup \{(x_i, d)\}, u' \in U^i_j, x_j \in Chld_i$.

Then gub_s is computed as $\min_{d\in D_i} gub_{a(s)\cup\{(x_i,d)\}}$. gub_s is used for two computations:

- gUB_i is updated so that $gUB_i \leftarrow \min(gub_s, gUB_i)$.
- As shown in the following, the condition of the optimal solution contains gub_s.

```
1  Main(){
2   Initialize().
3   until(forever){
4    until(receive loop is broken){ receive messages. }
5    if(S_i ≠ φ ∧ (sAct_i ∨ ¬cls_i)){ Maintenance(). } } }
6  Initialize(){
7   S_i ← φ. sAct_i ←false. sOpt_i ←false. sFin_i ←false.
8   sCls_i ←false. opt_i ←false. gUB_i ← ∞. cls_i ←false.
9   D_i^* ← φ.
10  foreach j s.t. x_j ∈ Chld_i{
11   (S_j^i, U_j^i, R_j^i, sFil_j^i, sChg_j^i)←(φ, φ, φ, false, false). }
12  if(x_i is root){
13   send (VALUES, {(φ, 0, 0)}, true, true, true, false) to i. } }
14 Receive(VALUES, S, sOpt, sChg, sCls, sFin){
15  S_i ← S. sCls_i ← sCls.
16  if(sChg){ sAct_i ←true. sOpt_i ← sOpt. sFin_i ← sFin.
17   cls_i ←false.
18   foreach j s.t. x_j ∈ Chld_i{
19    reset child states (S_j^i, U_j^i, R_j^i, sFil_j^i, sChg_j^i). } } }
20 Receive(COSTS, j, U){
21  if(compatible(U,S_j^i)){ merge U to U_j^i. } }
22 Receive(GUB, gUB){
23  gUB_i ← min(gUB_i, gUB). }
24 Maintenance(){
25  update (U_i, gUB_i) and test optimal condition.
26  if(opt_i ∧ (D_i has not been replaced by D_i^*)){
27   replace D_i by D_i^*.
28   if(x_i is root){ sFin_i ←true. }
29   foreach j s.t. x_j ∈ Chld_i{
30    reset child states (S_j^i, U_j^i, R_j^i, sFil_j^i, sChg_j^i). }
31   update (U_i, gUB_i) and test optimal condition. }
32  MaintainChildrensContexts().
33  cls_i ← sCls_i ∧ (all children's contexts are closed).
34  foreach j s.t. x_j ∈ Chld_i{
35   send(VALUES,S_j^i, opt_i, sChg_j^i,cls_i,sFin_i) to j. }
36  if(sAct_i ∨ ¬sCls_i){
37   if(x_i is not root){
38    send (COSTS,i,U_i) to j s.t. x_j = prnt_i. }
39   if(closed(U_i)){ sAct_i ←false. } }
40  foreach j s.t. x_j ∈ Chld_i{ send (GUB,gUB_i) to j. } }
41 MaintainChildrensContexts(){
42  foreach j s.t. x_j ∈ Chld_i{
43   update R_j^i. update current contexts S_j^i.
44   if(¬(contexts of j are active) ∧ ¬sFil_j^i){
45    set next contexts S_j^i.
46    if(S_j^i is last part of R_j^i){sFil_j^i ←true.}
47    sChg_j^i ←true. }
48   else{ sChg_j^i ←false. } } }
```

Figure 4: Multi-solution search with efficient methods

If $sOpt_i \wedge ((\exists u \in U_i, lb(u) = ub(u)) \vee ((\exists s \in S_i, gub_s = gUB_i)) \wedge \neg opt_i$, then an optimal solution is determined. Here, if $\exists s \in S_i, gub_s = gUB_i$, then d^* such that $gub_{a(s)\cup\{(x_i,d^*)\}} = gub_s$ is the optimal assignment of x_i. Otherwise, the computation of the optimal assignment is the same as the basic method.

closed(U_i): In addition to the original condition, lower bound glb_s of the global cost for element s of S_i is employed. The computation of glb_s is the same as the computation of gub_s except that the lower bounds are used instead of the upper bounds. Here, define condition $cond\_glb_s$ that depends on $sFin_i$:

$$cond\_glb_s \triangleq \begin{cases} glb_s > gUB_i & sFin_i \\ glb_s \geq gUB_i & \text{otherwise} \end{cases} \quad (4)$$

If $lb(u) = ub(u) \vee (cond\_glb_s$ where $a(s) = a(u))$ for all elements u of U_i, then the return value of closed(U_i) is true. Otherwise, the return value is false.

set next contexts S_j^i: When opt_i is false, then S_j^i is updated as follows. Let R_j^{i+} denote a subset of R_j^i. R_j^{i+} does not contain elements that have been already searched or pruned. S_j^i is updated as a subset of R_j^{i+}. If U_j^i does not contain u such that $a(s) = a(u)$ for an element s of S_j^i, then $(a(s), 0, \infty)$ is added to U_j^i. The following computation generates R_j^{i+} from R_j^i: First, for each element r of R_j^i, lower bound $glb_r = lb(r) + lb(u)$ of the cost is computed. Here, u such that $a(r) = a(u)$ is an element of U_j^i. Using condition $cond\_glb_r$ that resembles Expression 4, element r is evaluated. If $lb(u) = ub(u) \vee cond\_glb_r$, then r is not contained in R_j^{i+}. When opt_i is true, the computation is the same as the basic method.

update R_j^i: R_j^i is updated according to current S_i.

update current contexts S_j^i: For all elements s of S_j^i, $lb(s)$ and $ub(s)$ are updated based on current R_j^i.

all children's contexts are closed: If $lb(s) = ub(s)$ for all children j and all elements s of S_j^i, then the return value is true. Otherwise, the return value is false.

contexts of j are active: If $lb(u) \neq ub(u) \wedge \neg cond\_lb_s$ for at least one element s of S_j^i, the return value is true. Here, u such that $a(u) = a(s)$ is an element of U_j^i. Otherwise, the return value is false.

There are no modifications in 'merge U to U'' and COSTS message.

4.5 Extra messages

In the algorithms shown in Figure 3 and 4, identical messages are often repeatedly sent. Such redundant sending of messages can be blocked. However, for notification of a change of state, messages are sent when the VALUES messages are received (and when $sChg$ is true in Figure 4).

5 Correctness

The essential computation of the algorithms shown in this paper resembles conventional algorithms. The bottom-up computation of optimal cost and the top-down computation of global cost depend on current contexts. When the current contexts have not changed, the boundaries of the cost values are monotonously narrowed. The current contexts of each agent eventually reflect the last decision of the ancestor agents. Therefore, if pruning is not applied, the boundaries converge to an optimal value. Pruning does not destroy the boundaries because it only skips

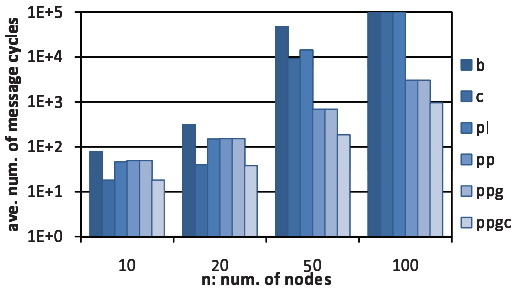


Figure 5: max-csp-gcl ($l = 1.5, L = 27$)

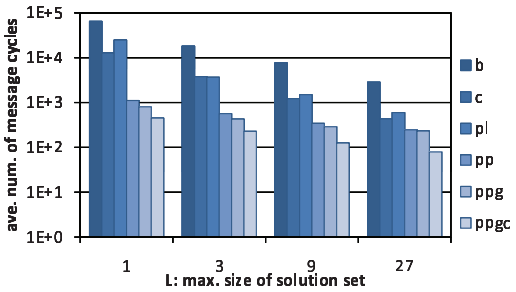


Figure 6: max-csp-gcl ($n = 20, l = 2$)

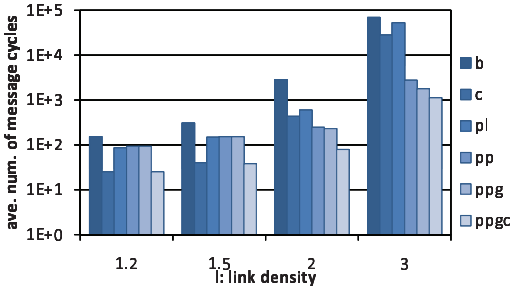


Figure 7: max-csp-gcl ($n = 20, L = 27$)

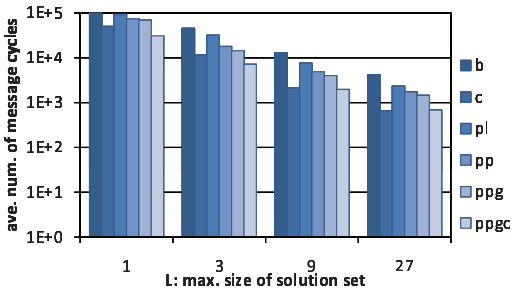


Figure 8: cop ($n = 20, l = 2$)

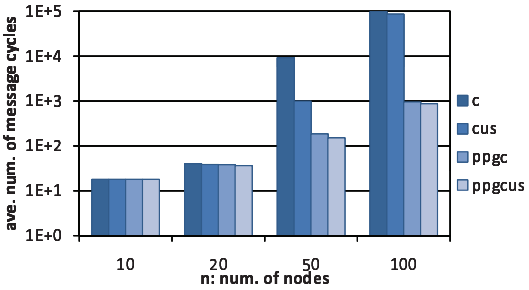


Figure 9: max-csp-gcl ($l = 1.5, L = 27$)

the search. Moreover, pruning is only judged based on the current states of each agent. The upper bound of the global cost value is monotonously decreased and immediately propagated. Switching of the pruning condition is also immediately propagated using $sFin_i$. Therefore, situations where pruning is excessive eventually disappear and the search processing converges.

6 Evaluation

As the first result, we experimentally evaluated the effects of our proposed methods, which were applied to two types of problems that consist of n ternary variables and $l \cdot n$ binary constraints/functions. Similar classes of the problems are used to evaluate DCOP solvers in several studies. Here parameters $l = 1.2, 1.5, 2, 3$ were used.

max-csp-gcl: the problems resemble maximum constraint satisfaction problems for vertex coloring. If $x_i = x_j$, then the value of $f_{i,j}(x_i, x_j)$ is 1. Otherwise, the cost value is 0.

cop: $f_{i,j}(x_i, x_j)$ takes integer values between 0 and 10. The values are randomly set based on uniform distribution.

The results are average of twenty instances. Actually, the results are significantly affected by the instances whose induced width is large. b (Figure 3), c (b with reusing U^i_j), ppgc (Figure 4), ppg (ppgc without reusing of U^i_j), pp (ppg without gUB_i in the test of optimal condition), and pl (pp without top-down costs) were compared. Solutions are expanded in a depth-first manner based on a fixed order of variables/values. Maximum number $L = 1, 3, 9, 27$ of $|S^i_j|$ and maximum size $K = L \cdot |D_i|$ of $|U^i_j|$ are used. The number of message cycles was evaluated. In each message cycle, agents receive messages from their receive-queue, perform its processing and send messages to their send-queue if necessary. Then the messages in the queues are exchanged. The experiment was stopped at the 10^5 cycle. In that case, the number of message cycles was considered to be 10^5.

The number of message cycles in max-csp-gcl is shown. Figure 5 shows the case of $l = 1.5, L = 27$. In the problems of $n = 50, 100$, both pruning and reusing U^i_j are effective. Induced-width tends to grow with the increase of n in these problems. Therefore, the effect of c, which only employs the reuse of U^i_j, is small when n is relatively large. Figure 6 shows the case of $n = 20, l = 2$. In these results, the increment of L is relatively effective in the methods that employ pruning. Figure 7 shows the case of $n = 20, L = 27$. The results resemble Figure 5 because the induced width tends to grow with the increase of l. When pruning methods of pl, pp and ppg are effective, they complementarily reduce the number of message cycles. The number of message cycles in cop is shown in Figure 8. Increasing L reduced the number of message cycles. However, the difference of ppgc and c is relatively small even if dependence on pruning is large ($L = 1, 3$). In the results, only reusing U^i_j is relatively effective. Actually, ppgc is less effective than c in several results. This drawback might be caused by the pruning that affects the order

Table 1: Size of pseudo-trees

l	1.2			1.5			2			3		
n	td	tb	sz	td	tb	sz	td	tb	sz	td	tb	sz
10	5	1.4	14	6	1.2	32	7	1.2	89	9	1.0	729
20	9	1.5	41	11	1.4	192	13	1.3	2138	15	1.2	84637
50	18	1.6	1334	21	1.5	$1.6_{\times 10^5}$	29	1.4	$1.5_{\times 10^7}$	34	1.3	$1.4_{\times 10^{11}}$
100	30	1.7	32295	42	1.5	$5.9_{\times 10^9}$	51	1.4	$9.7_{\times 10^{14}}$	64	1.3	$2.5_{\times 10^{21}}$

td: depth, tb: branching factor without leaf, sz: $\max_{x_i} \prod_{k \in Ast_i} |D_k|$

Table 2: Number of messages per message cycle

(max-csp-gcl, $n = 50, l = 1.5, L = 9$)

alg.	num. of act. pairs	num. of messages			
		all	VALUES	COSTS	GUB
b	9.3	9.3	4.4	5.0	0
c	3.0	3.0	1.3	1.6	0
pl	4.0	4.1	1.4	2.5	0.1
pp	7.8	8.2	4.4	3.0	0.8
ppg	7.8	8.2	4.4	3.0	0.8
ppgc	6.9	8.1	3.5	2.3	2.3

of the cached elements.

The size of the pseudo-trees is shown in Table 1. The induced width of the pseudo-trees grow with the increases of l and n. Therefore, maximum size sz of the partial problems increases. Table 2 shows the number of messages per message cycle. The number of pairs of agents (source, destination) that transferred at least one message in one message cycle is also shown. When no messages are received, the agents are in quiescence. Redundant sending of messages is blocked, as shown in 4.5. Therefore, the frequency of message communication is relatively small.

In the algorithms shown in Sections 3 and 4, current set S_j^i of the contexts is updated to subsequent contexts when all boundaries of all contexts are closed. However, when the boundaries of one element s of S_j^i are closed, s can be immediately replaced with the next context. We applied the above methods to c and ppgc. Figure 9 shows the results of the methods. cus and ppgcus are the modified algorithms. The number of message cycles of c was relatively decreased in several cases. On the other hand, for ppgc that employs pruning, the effect of the modification was small.

7 Conclusion

In this study, we present the basic algorithms of memory-bounded solvers that employ multiple search points. Such methods resemble the execution of overlapped multiple search processes. In search processing, the solver recomposes the sets of solutions that are currently expanded. As a result, iterative search that depends on message communication is reduced. Basically, the proposed methods are generalizations of previous studies. To clarify the essence of the processing, we obviously used several fundamental computations in the algorithms. The solution spaces of the sub-problems are considered when sub-solutions are expanded. The boundaries of global cost values are exactly and separately computed. Such methods can be considered as the basis of solvers that employ mixed search strategies. Several experimental results show that the proposed method efficiently works with pruning and reusing results. Detailed investigation of the trade-off between computation and communication, multiple search strategies, and other additional efficient methods are future works.

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